

# On influence of gravitational waves on circular moving particles

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We investigate the influence of a gravitational wave background on particles in circular motion. We are especially interested in waves leading to stationary orbits. We limit this consideration to circular orbits perpendicular to the incidence direction. A background of gravitational waves creates some kind of uncertainty.

## I. INTRODUCTION

One hundred years after Albert Einstein predicted the existence of gravitational waves and after decades of searches, LIGO has succeeded to detect gravitational waves [1]. Obviously, they originated from a merger of two stellar-mass black holes. With sizes of the interferometer arms of 4 km LIGO is most sensitive in the frequency band 100–300 Hz. This success of LIGO could support the idea of the existence of gravitation waves in very different frequency ranges. In the Big Bang and in the later history of the universe a background of gravitational waves could have been produced, still today influencing elementary particles at the atomic scale, as one could speculate. Especially interesting would be an influence on electrons in hydrogen employing Bohr's atomic model. But apart from this special example the formalism could be valid quite generally for particles moving in a central force field.

Tiny ripples in spacetime curvature propagate as waves with the speed of light as was shown by Einstein himself. They lead to periodic oscillations in the distance of test particles, as nicely expounded in many books and review articles, e.g. in [2–4, 7–9]. An overview of detection of gravitational waves can be read in [10]. Therein it is as well described how gravitational waves arise from general relativity. In [6] a foundational review about gravitational waves is given and a critical review of the standard linear approach of the theory is depicted.

The aim of this paper is to investigate particles moving in a central force field under the influence of gravitational waves. To achieve this goal we have to modify the equation of geodesic deviation including additional force terms. We are especially interested in uncertainties of the orbits and conditions for stationary orbits.

The paper is organized as follows: In chapter II we repeat the results obtained for distances of test particles in the presence of plane gravitational waves. Afterwards, in chapter III, we discuss an extension of the problem

by considering the influence of such gravitational waves on circular moving particles. In chapter IV results of the previous chapter are graphically displayed and discussed. Finally in chapter V conclusions are drawn.

## II. MOTION OF TEST PARTICLES

In order to obtain a coordinate independent measure of the wave's influence, the relative motion of two nearby particles can be considered, see e.g. in [3]. It can be described by the geodesic equation in four-dimensional space-time  $x^\mu = (ct, x, y, z)$ . The four-velocity is given by  $U^\mu = \frac{dx^\mu}{d\tau}$  and the distance vector  $S^\mu$  is a solution of the differential equation

$$\frac{D^2}{d\tau^2} S^\mu = S^\sigma U^\nu U^\rho R^\mu{}_{\nu\rho\sigma} \quad (1)$$

where  $\eta^{\mu\lambda} = \text{diag}(-1, 1, 1, 1)$  is the Minkowski metric in the flat background. The double differential  $\frac{D^2}{d\tau^2}$  points to the parallel transport in general relativity and the differential of the proper time is given by  $d\tau = \frac{1}{\gamma} dt$  with  $\gamma = \frac{1}{\sqrt{1-\beta^2}}$  and  $\beta = \frac{v}{c}$ .

In curved space-time the linearized Riemann-tensor  $R_{\mu\nu\rho\sigma}$  is given by

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}(\partial_\rho\partial_\nu h_{\mu\sigma} + \partial_\sigma\partial_\mu h_{\nu\rho} - \partial_\sigma\partial_\nu h_{\mu\rho} - \partial_\rho\partial_\mu h_{\nu\sigma}). \quad (2)$$

Here the metric  $g_{\mu\nu}$  has been approximated by  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$  with  $|h_{\mu\nu}| \ll 1$ .

For a gravitational wave  $h_{\mu\nu}$ , propagating in  $z$ -direction with the velocity of light  $c$ , one obtains harmonic oscillations which for simplicity can be written as (the real part of) plane waves [2–4, 7–9]

$$h_{\mu\nu} = C_{\mu\nu} E \quad \text{with} \quad E := e^{i(kz - \omega_g t)} \quad (3)$$

where  $\omega_g = ck$  is the frequency of the gravitational wave ( $k$  is the wave number). The constant quantities  $C_{\mu\nu} \ll 1$  form a symmetric  $(0, 2)$  tensor with  $C_{0\nu} = C_{3\nu} = 0$  and

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$C_{22} = -C_{11}$  as well as  $C_{12} = C_{21}$ . They generate a time-varying quadrupole deformation. The two parameters  $C_{11}$  and  $C_{12}$  are sufficient to describe any quadrupole deformation in the xy-plane.

Because the Riemann-tensor is first order, the corrections to  $U^\nu$  may be ignored, and, for slowly moving particles ( $\tau = x^0 = ct$ ), we have  $U^\nu = (1, 0, 0, 0)$ . As a consequence  $R_{\mu 00\sigma} = \frac{1}{2}\partial_0^2 h_{\mu\sigma}$  and the geodesic equation (1) becomes

$$\frac{\partial^2}{\partial t^2} S^\mu \stackrel{(1)}{=} \frac{1}{2} S^\sigma \frac{\partial^2}{\partial t^2} h^\mu{}_\sigma. \quad (4)$$

$C_{11}$  and  $C_{12}$  differ only by a  $\pi/4$ -rotation in the xy-plane. Therefore we can choose without loss of generality  $C_{12} = 0$  resulting in

$$\left. \begin{aligned} \partial_0^2 S^1 &= \frac{1}{2} S^1 \partial_0^2 h_{11}, \\ \partial_0^2 S^2 &= -\frac{1}{2} S^2 \partial_0^2 h_{11} \end{aligned} \right\} \text{ with } \begin{cases} h_{11} \stackrel{(3)}{=} C_{11} E, \\ E := e^{i(kz - \omega_g t)}. \end{cases} \quad (5)$$

For  $C_{11} \ll 1$  the solutions of these equations are given in refs. [2–4, 7–9] by

$$\begin{aligned} S^1 &\approx S_c^1(t) = S_c^1(0) \left[ 1 + \frac{C_{11}}{2} E \right], \\ S^2 &\approx S_c^2(t) = S_c^2(0) \left[ 1 - \frac{C_{11}}{2} E \right]. \end{aligned} \quad (6)$$

$S_c^1(t)$  and  $S_c^2(t)$  are the coordinates of the separation vector in the xy-plane. The oscillations in x and y coordinates are  $180^\circ$  out of phase and lead therefore to linear oscillations around the vector  $(S_c^1(0), S_c^2(0))$ , as shown in Fig. 1. To increase the visibility we have chosen an unre-

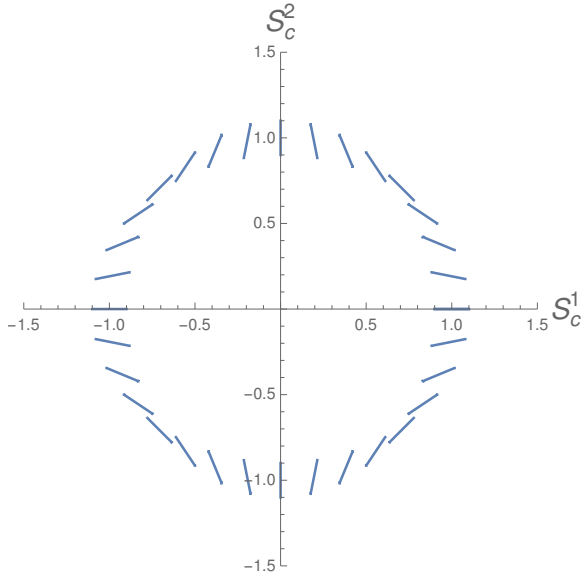


FIG. 1. Linear oscillations of  $(S_c^1(t), S_c^2(t))$  with time  $t$  for various values of  $S_c^1(0)$  and  $S_c^2(0)$  with  $\sqrt{[S_c^1(0)]^2 + [S_c^2(0)]^2} = 1$  and  $C_{11} = 0.2$ .

alistic large value for  $C_{11}$ . The functions in Eq. (6) fulfil

Eq. (5) up to order  $C_{11}^2$  only

$$\begin{aligned} -\omega_g^2 S_c^1(0) \frac{C_{11}}{2} E \stackrel{(5)}{\approx} -\omega_g^2 S_c^1(0) \frac{C_{11}}{2} E \left( 1 + \frac{C_{11}}{2} E \right) \\ \rightarrow 1 \approx 1 + \frac{C_{11}}{2} E. \end{aligned} \quad (7)$$

### III. CIRCULAR MOTION WITH CONSTANT ANGULAR VELOCITY

Let us consider a light particle rotating on a circle with radius  $r$  with constant angular velocity  $\omega$  around a heavy particle, like in Bohr's model for the hydrogen atom. The distance vector from the heavy to the light particle we write as

$$r^\mu = (ct, r \cos \varphi, r \sin \varphi, 0) \text{ where } \varphi = \omega t \quad (8)$$

and the four-velocity as

$$U^\mu = \gamma (1, -\beta \sin \varphi, \beta \cos \varphi, 0), \quad \beta = \frac{r\omega}{c}, \quad (9)$$

with  $U^\mu U_\mu = -1$ .

We go back to Eq. (1) taking into account the geometrical term  $S^\sigma U^\nu U^\rho R^\mu{}_{\nu\rho\sigma}$  representing the curved space-time due to gravitational waves. By the same argument as in section II we choose again  $C_{12} = 0$ . Therefore, in expression (3) for  $h_{\mu\nu}$  only  $h_{11}$  and  $h_{22} = -h_{11}$  are non-vanishing. According to Eq. (3)  $h_{\mu\nu}$  depends only on  $z$  and  $t$ . The summation in the differential equation (1) extends over  $\nu, \rho, \sigma \in \{0, 1, 2, 3\}$ . As a consequence of the geometrical assumptions — wave propagating in z-direction and particle rotating in xy-plane — we expect  $S_3 = U_3 = 0$ . Consequently in Eq. (2) only time derivatives of  $h_{\mu\nu}$  matter. Therefore, the geometrical term  $S^\sigma U^\nu U^\rho R^\mu{}_{\nu\rho\sigma}$  simplifies drastically and gives for  $\nu = 0, 1, 2$

$$\begin{aligned} S^\sigma U^\nu U^\rho R^0{}_{\nu\rho\sigma} &\stackrel{(2)}{=} \\ &= -\partial_0^2 \frac{h_{11}}{2} [S^0(U^1 U^1 - U^2 U^2) - S^1 U^0 U^1 + S^2 U^0 U^2], \\ S^\sigma U^\nu U^\rho R^1{}_{\nu\rho\sigma} &\stackrel{(2)}{=} \\ &= \partial_0^2 \frac{h_{11}}{2} [-S^0 U^0 U^1 + S^1 U^0 U^0], \\ S^\sigma U^\nu U^\rho R^2{}_{\nu\rho\sigma} &\stackrel{(2)}{=} \\ &= \partial_0^2 \frac{h_{11}}{2} [S^0 U^0 U^2 - S^2 U^0 U^0], \end{aligned} \quad (10)$$

with  $\partial_0^2 \frac{h_{11}}{2}$  of first order in  $C_{11}$

$$\partial_0^2 \frac{h_{11}}{2} = -\omega_g^2 E \frac{C_{11}}{2} \quad \text{with} \quad E := e^{i(kz - \omega_g t)} \quad (11)$$

We use now an ansatz in the spirit of the approximate

solution (6)

$$\begin{aligned} S^1 &\approx r \left[ 1 + \frac{C_{11}}{2} E \right] \cos \varphi \equiv S_r^1 \cos \varphi, \\ S^2 &\approx r \left[ 1 - \frac{C_{11}}{2} E \right] \sin \varphi \equiv S_r^2 \sin \varphi, \\ S^0 &= S^3 = 0. \end{aligned} \quad (12)$$

with  $\varphi = \omega t$  and  $E = e^{i(kz - \omega_g t)}$ .

In Eq. (10) the velocities appear in terms of order  $C_{11}$ . Up to this order of accuracy, we can continue to use the four-velocity Eq. (9). In the acceleration term on the left side of Eq. (1) we use

$$\frac{D^2}{d\tau^2} S^\mu = \gamma^2 \partial_0^2 S^\mu \quad (13)$$

and

$$\begin{aligned} \partial_0^2 S^1 &= (\partial_0^2 S_r^1) \cos \varphi - 2(\partial_0 S_r^1) \omega \sin \varphi - S_r^1 \omega^2 \cos \varphi, \\ \partial_0^2 S^2 &= (\partial_0^2 S_r^2) \sin \varphi + 2(\partial_0 S_r^2) \omega \cos \varphi - S_r^2 \omega^2 \sin \varphi. \end{aligned} \quad (14)$$

The last terms in these equations represent the central force keeping the light particle on a circle. The middle terms reflect the contribution of the Coriolis force. The first terms on the right side take into account the influence of the gravitational wave described in Eqs. (10). Using Eq. (13) this implicates the generalised equations of motion

$$\partial_0^2 S^\mu = S^\sigma \frac{U^\nu}{\gamma} \frac{U^\rho}{\gamma} R^\mu_{\nu\rho\sigma} + 2(0, \vec{\omega} \times \vec{v}_r)^\mu - \omega^2 S^\mu, \quad (15)$$

where  $\vec{\omega} = (0, 0, \omega)$  and  $\vec{v}_r$  is the radial velocity

$$\vec{v}_r = ((\partial_0 S_r^1) \cos \varphi, (\partial_0 S_r^2) \sin \varphi, 0). \quad (16)$$

We can now check the accuracy of ansatz (12). We insert Eq. (14) into the left hand side of Eq. (15) and get for  $\mu = 1$

$$\text{lhs}^1 \stackrel{(15)}{=} -\omega_g^2 E \frac{C_{11}}{2} r \cos \varphi - 2(\partial_0 S_r^1) \omega \sin \varphi - \omega^2 S^1. \quad (17)$$

With Eq. (10) we get for the right hand side

$$\text{rhs}^1 \stackrel{(15)}{=} -\omega_g^2 E \frac{C_{11}}{2} S^1 + 2 \vec{\omega} \times \vec{v}_r^1 - \omega^2 S^1. \quad (18)$$

The two expressions agree up to order  $C_{11}$ , to the same order as the well-known solution (6) of Eq. (5)

$$\text{lhs}^1 \approx \text{rhs}^1 \quad (19)$$

For  $\mu = 2$  we get

$$\begin{aligned} \omega_g^2 E \frac{C_{11}}{2} r \sin \varphi + 2(\partial_0 S_r^2) \omega \cos \varphi - \omega^2 S^2 \\ \approx +\omega_g^2 E \frac{C_{11}}{2} S^2 + 2 \vec{\omega} \times \vec{v}_r^2 - \omega^2 S^2. \end{aligned} \quad (20)$$

which is valid to the same accuracy.

The corresponding comparison for  $\partial_0^2 S^0$  reads

$$\begin{aligned} \partial_0^2 S^0 &= 0 \\ &\approx \omega_g^2 E \frac{C_{11}}{2} \beta \{S^1 \sin \varphi + S^2 \cos \varphi\} \approx \\ &\approx \omega_g^2 E \frac{C_{11}}{2} \beta r \sin(2\varphi). \end{aligned} \quad (21)$$

This starts to disagree at order  $\beta C_{11}$  which is suppressed by  $\beta$  and  $C_{11}$ .

Therefore one can conclude that  $S^\mu$  from Eq. (12) are the proper solutions of Eq. (15).

#### IV. RESULTS AND DISCUSSION

With the expressions (12) for  $S^\mu$  we are now able to display figures of the orbits. To allow for better visibility we choose the amplitudes of the quadrupole oscillations in most diagrams unrealistically large,  $C_{11}/2 = 0.1$ . Only in Fig. 5 and Fig. 9 we choose slightly smaller values. In Figs. 2-5 the orbits close after one revolution  $T$  due to the integer frequency ratios  $\frac{\omega_g}{\omega}$ . With  $\frac{\omega_g}{\omega} = 1.25$  the path does not close yet after 2 revolutions in Fig. 6, but it would after 4. In Fig. 7, 8 and 9 we choose  $\frac{\omega_g}{\omega} = 2.1$  and 1.02 respectively and need 10 and 50 revolutions to get a closed paths. In Figs. 8 and 9 we compare with different values for the amplitude  $C_{11}$ . In the diagrams 10 and 11 we modify the synchronisation of oscillation and rotational motion by choosing an additional phase,  $kz = \pi/4$ , leading to a rotation of the diagram compared to the figures 2 and 3.

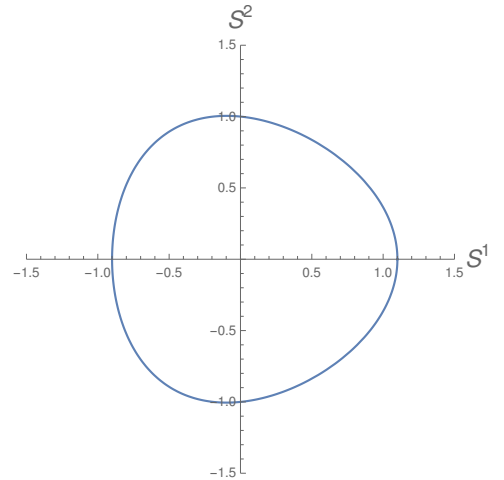


FIG. 2.  $\frac{C_{11}}{2} = 0.1$ ,  $kz = 0$ ,  $\frac{\omega_g}{\omega} = 1$ ,  $t = T$ .

These examples clearly reveal the influence of gravitational waves on circulating particles in the space-time structure of general relativity.

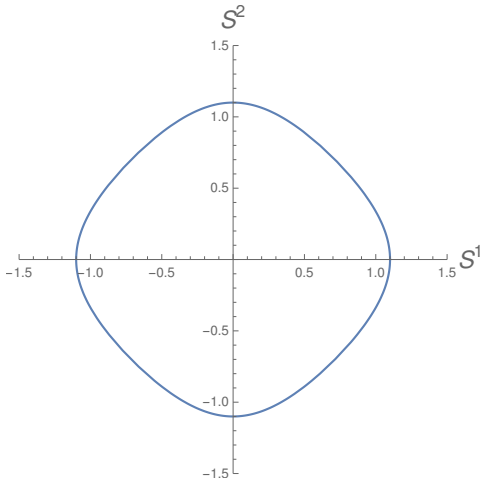


FIG. 3.  $\frac{C_{11}}{2} = 0.1$ ,  $kz = 0$ ,  $\frac{\omega_g}{\omega} = 2$ ,  $t = T$ .

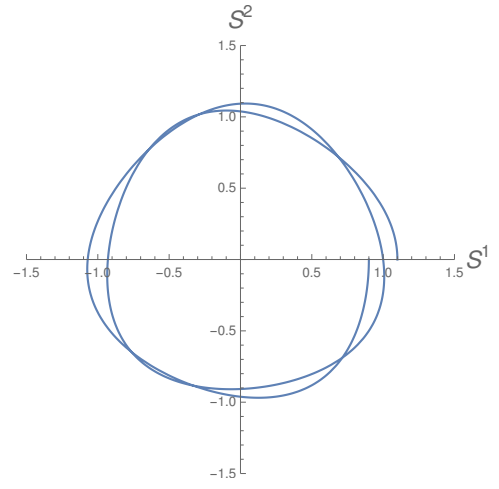


FIG. 6.  $\frac{C_{11}}{2} = 0.1$ ,  $kz = 0$ ,  $\frac{\omega_g}{\omega} = 1.25$ ,  $t = 2 T$ .

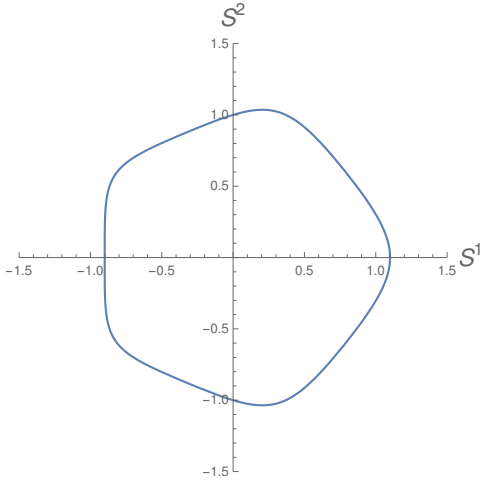


FIG. 4.  $\frac{C_{11}}{2} = 0.1$ ,  $kz = 0$ ,  $\frac{\omega_g}{\omega} = 3$ ,  $t = T$ .

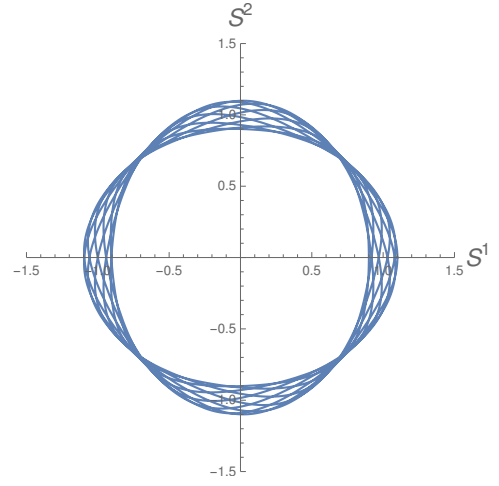


FIG. 7.  $\frac{C_{11}}{2} = 0.1$ ,  $kz = 0$ ,  $\frac{\omega_g}{\omega} = 2.1$ ,  $t = 10 T$ .

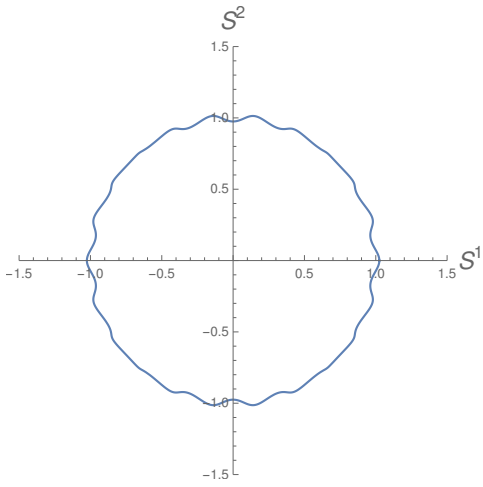


FIG. 5.  $\frac{C_{11}}{2} = 0.025$ ,  $kz = 0$ ,  $\frac{\omega_g}{\omega} = 20$ ,  $t = T$ .

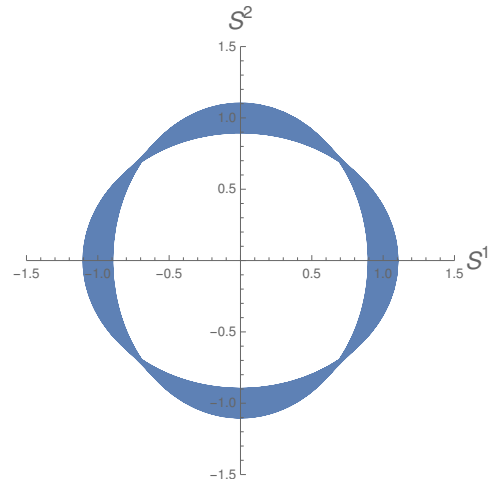


FIG. 8.  $\frac{C_{11}}{2} = 0.1$ ,  $kz = 0$ ,  $\frac{\omega_g}{\omega} = 1.02$ ,  $t = 50 T$ .

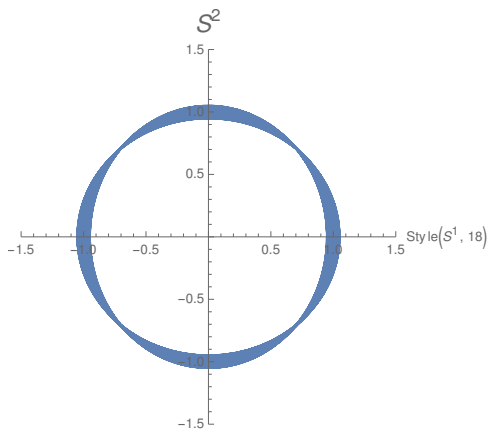


FIG. 9.  $\frac{C_{11}}{2} = 0.05$ ,  $kz = 0.$ ,  $\frac{\omega_g}{\omega} = 1.02$ ,  $t = 50 T$ .

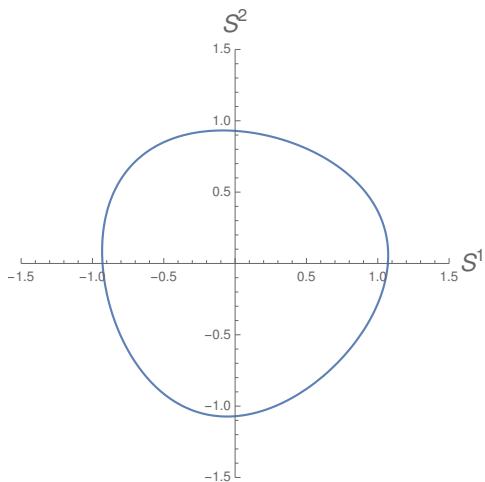


FIG. 10.  $\frac{C_{11}}{2} = 0.1$ ,  $kz = \pi/4$ ,  $\frac{\omega_g}{\omega} = 1$ ,  $t = T$ .

## V. CONCLUSION

In this work we attempted to investigate the influence of gravitational waves on particles circulating in a central force field. We especially concentrated on periodic orbits and their perturbations.

After the introduction we recapitulated in chapter II the very well known approximate solution for slowly moving particles subjected to gravitational waves with frequency  $\omega_g$ . In chapter III we expanded the problem to particles circulating with frequency  $\omega$  in a central force field. To get a precision of the same order as for slowly moving test particles in refs. [2–4, 7–9] we had to take into account the centrifugal force and the Coriolis force. Chapter IV is devoted to the graphical representation of

the results. Besides the amplitude of the gravitational wave the ratio  $\omega_g/\omega$  plays an important role. For integer values we get periodic orbits. For non-integer values of  $\omega_g/\omega$  the paths are disturbed. For the shape of these perturbations we have to take into consideration that incident gravitational waves will have different po-

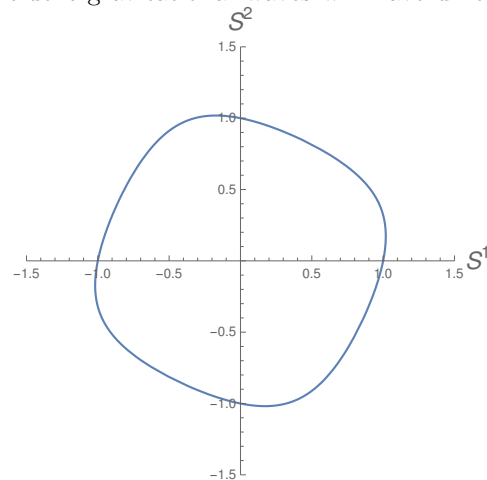


FIG. 11.  $\frac{C_{11}}{2} = 0.1$ ,  $kz = \pi/4$  and  $\frac{\omega_g}{\omega} = 2$ ,  $t = T$ .

larisations and therefore different positions of the nodes. In any case a background of gravitational waves creates a kind of uncertainty in rotating systems.

The idea to this work is inspired by the silicon oil drop experiment of Yves Couder and his group. This is the only experiment, we know, which could give some idea why we can describe nature perfectly by quantum mechanics. A silicon droplet bouncing on a vibrating fluid bath creates waves interfering with the background field [5]. The resonant interaction of particle and field creates a wave field guiding moving droplets.

Thinking about the nature of a subquantum medium which could guide elementary particles we observe the importance of the Compton wave length which is related to the mass of particles. A natural type of background waves which could feel the mass of particles are gravitational waves. Waves which are not in resonance would lead to disturbances and to uncertainties in the position and momentum as we have seen in the investigation presented above. To get closer to quantum mechanics it would be necessary that the size of these displacements is related to the Compton wavelength. In the above linear treatment we find closed orbits for any integer ratio  $\omega_g/\omega$ . This does not agree with Bohr's quantisation condition. It would be interesting to take into account the non-linear terms of Einstein's equation and to investigate whether we get a relation to Bohr's quantisation condition.

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